

# Squeezed light from spin squeezed atoms

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We propose to produce pulses of strongly squeezed light by Raman scattering of a strong laser pulse on a spin squeezed atomic sample. We prove that the emission is restricted to a single field mode which perfectly inherits the quantum correlations of the atomic system.

PACS numbers: 03.75.Fi, 05.30.Je

Squeezed light and entangled beams of light can be used to probe matter and to study mechanical motion with better resolution than classical light [1]. Entangled photon sources can be used for lithography with a resolution below the optical wavelength [2, 3], and the active field of quantum information profits from the development of non-classical light sources [4, 5, 6]. Non-linear crystals in optical parameter oscillators and laser diodes with suitable feed-back have been the work horses in most experiments on non-classical light. As a figure of merit for the degree of non-classicality of these light sources, one may refer to the noise suppression observed in direct or homodyne photon detection measurements. Compared to classical sources it has so far been possible to reduce the noise (variance) by about one order of magnitude. In order to make a significant difference in practical applications, further noise reduction is really necessary.

It has been proposed that the large non-linearity and low absorption in resonant Raman systems can lead to ideal four-wave mixing and substantial squeezing [7, 8, 9]. As an alternative approach we propose in this Letter to use spin squeezed ensembles of atoms as sources of squeezed light. Atoms can be entangled in such a way that the fluctuations in occupancies of different internal states are significantly suppressed. This phenomenon is referred to as spin squeezing, because a two level atom can be described formally as a spin 1/2 particle, and the interest in spin squeezed states arose already a long time ago in connection with ultra-precise spectroscopy and atomic clocks [10]. Squeezing of spins was originally believed to be very complicated, but recent proposals based on quantum non-demolition measurements of atomic populations [11], on coherent interactions in Bose-Einstein condensates [12, 13], and on interactions between laser excited atoms [14] have changed this impression and suggested that really significant spin squeezing is achievable. The main purpose of the Letter is to demonstrate that the atomic quantum correlations can be perfectly transferred to the field. This result is readily obtained within a simplified model where both atoms and field are described by single harmonic oscillators [15] but we show that correlations in the atoms can be mapped perfectly on the

field also in an a priori multimode situation.

The emission of light is treated by a simple generalization of the theory of stimulated Raman scattering [16, 17, 18] to and this part of our proposal can be analyzed without specifying the model for spin squeezing. Our ensemble of two-state atoms is assumed to be strongly elongated and it is treated in a 1D approximation. It is illuminated by a strong laser field  $E_s$  propagating along the  $z$ -axis of the system. This opens up a channel for an atom in the  $b$  state to go to the  $a$  state by absorbing a photon of frequency  $\omega_s$  and emitting a photon of frequency  $\omega_q \sim \omega_s + \omega_{ba}$ . As a result, a field at this frequency builds up and propagates through the sample. This process is described by the following coupled set of equations for the atomic and field operators

$$\begin{aligned} \frac{\partial}{\partial t} [\hat{\psi}_b^\dagger(z, t) \hat{\psi}_a(z, t)] &= -i\kappa_1^* E_s^*(z, t) \times \\ &\times \left( \hat{\psi}_a^\dagger(z, t) \hat{\psi}_a(z, t) - \hat{\psi}_b^\dagger(z, t) \hat{\psi}_b(z, t) \right) \hat{E}_q(z, t) \\ \left( \frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) \hat{E}_q(z, t) &= -i\kappa_2 \hat{\psi}_b^\dagger(z, t) \hat{\psi}_a(z, t) E_s(z, t). \end{aligned} \quad (1)$$

$$(2)$$

where  $\kappa_1 = \sum_i \mu_{ai} \mu_{bi} / (\hbar^2 \Delta_i)$  and  $\kappa_2 = 2\pi \hbar \omega_q \kappa_1 / c$ .  $\mu_{ji}$  are dipole moments of the atomic transitions and  $\Delta_i$  are the (large) detunings with respect to intermediate levels, see Fig. 1.  $\hat{\psi}_a(z, t)$ ,  $\hat{\psi}_a^\dagger(z, t)$  and  $\hat{\psi}_b(z, t)$ ,  $\hat{\psi}_b^\dagger(z, t)$  are annihilation and creation operators for atoms in states  $a$  and  $b$ ;  $\hat{\psi}_b^\dagger(z, t) \hat{\psi}_a(z, t)$  is the positive frequency part of the atomic dipole operator, taking into account the atomic density at position  $z$  in the ensemble. In Eq. (1) we have assumed that there is no dephasing of the  $ab$  coherence and we have assumed the validity of the slowly varying envelope approximation for the emitted field in Eq. (2).

We restrict our analysis to the case where the atoms are almost entirely in the  $a$  state when the  $E_s$  field is applied. This implies that the population difference appearing in Eq. (1) can be replaced by the density of atoms  $n(z)$ . This density we represent as a  $c$ -number throughout the duration of the output coupling. This allows us to define a dipole operator "per atom" by  $\hat{\psi}_b^\dagger(z, t) \hat{\psi}_a(z, t) = n(z) \hat{Q}(z, t)$  and we obtain the linear

operator equations

$$\frac{\partial}{\partial t} \hat{Q}(z, t) = -i\kappa_1^* E_s^*(z, t) \hat{E}_q(z, t) \quad (3)$$

$$\left( \frac{\partial}{\partial z} + \frac{1}{c} \frac{\partial}{\partial t} \right) \hat{E}_q(z, t) = -i\kappa_2 n(z) E_s(z, t) \hat{Q}(z, t). \quad (4)$$

If we generalize the analysis in [16, 17, 18] to inhomogeneous media, we can solve Eqs. (3) and (4) analytically in terms of the input field  $\hat{E}_q(0, t)$  at the entrance face of the sample at  $z = 0$  and the initial position dependent atomic polarization  $\hat{Q}(z, 0)$ . In particular, for the field operator we get

$$\begin{aligned} \hat{E}_q(z, \tau) = & \hat{E}_q(0, \tau) - i\kappa_2 E_s(\tau) \int_0^z \psi_b^\dagger(z', 0) \psi_a(z', 0) \times \\ & \times J_0 \left( 2\sqrt{a(\tau) \int_{z'}^z n(z'') dz''} \right) dz' \end{aligned} \quad (5)$$

where the new time coordinate is  $\tau \equiv t - z/c$ ,  $J_0(\cdot)$  is a Bessel function of the first kind, and  $a(\tau) = \kappa_1^* \kappa_2 \int_0^\tau |E_s(\tau')|^2 d\tau'$ .

The expression (5) must be evaluated at the position  $z = L$  of a detector outside the atomic sample. In the absence of atoms, the field equals the incident quantum vacuum field  $\hat{E}_q(0, \tau)$ . The atomic sample is able to replace the vacuum with an entirely different field. In order to analyze the quantum properties of  $\hat{E}_q$  it is convenient to imagine a time integrated homodyne detection at the detector. By choosing the temporal form of the strong local oscillator field in this detection we select a certain spatio-temporal mode of the field represented by the field operator

$$\hat{a} = \sqrt{\frac{c}{2\pi\hbar\omega_q}} \int_0^\infty \mathcal{E}^*(\tau) \hat{E}_q(L, \tau) d\tau \quad (6)$$

where  $\int |\mathcal{E}|^2 d\tau = 1$ . In choosing  $\mathcal{E}(\tau)$  we should seek to ensure that  $\hat{a}$  is a mapping of the precise collective operator of the atomic sample that is known to be squeezed. Mathematically, it is easy to show that in order to probe  $\int h(z') \hat{Q}(z', 0) dz'$  we should choose  $\mathcal{E}(\tau)$  as the normalized solution of Eqs. (3,4) with the initial condition  $n(z') \hat{Q}(z', 0)$  replaced by  $h(z')$ . Physically, this reflects that the question of mode matching coincides with the problem of identifying the classical field radiated by a classical dipole distribution. In particular, if the uniform integral of atomic operators  $\int \psi_b^\dagger(z', 0) \psi_a(z', 0) dz'$  is squeezed, mode matching is accomplished by taking  $\mathcal{E}(\tau)$  to be the (normalized) solution to the classical Raman scattering problem. In that case we get the mapping

$$\hat{a} = \frac{1}{\sqrt{N}} J_- \quad (7)$$

where  $\hat{J}_- \equiv \int \hat{\psi}_b^\dagger(z, t) \hat{\psi}_a(z, t) dz$ , and  $N$  is the total number of atoms.

It is a natural concern, whether a 3D analysis will preserve the possibility to couple to only a single spatio-temporal field mode. The issue has been addressed in the classical case, where a diagonalization of the first order coherence function (Karhunen-Loeve transformation) of the field indeed shows that a single mode dominates the output, provided the Fresnel number of the (active part of the) atomic sample is of the order of unity [17]. Choosing the experimental parameters accordingly we thus expect this result to apply also for the quantum field in three dimensions.

We recall that several schemes now exist for the generation of non-classical states of atomic spins. The above analytical treatment is general and independent of the method of spin squeezing, and as shown by Eq.(7) (valid only if the majority of atoms occupy the state  $a$ ), the atomic state is transferred perfectly to the light field. Mean values and variances for the field observables are therefore explicitly known, and, *e.g.*, the orders of magnitude squeezing derived in [12, 13] apply to the emitted light pulse. It may be useful, however, to relate the emitted quantum field directly to the dynamical variables in the spin squeezing process. This may provide further insight in the origin of squeezing; it may provide a useful tool for the application of the squeezed light as input to another quantum system [19, 20]; and, it may be useful to analyze processes where the spin squeezing and the light emission occurs simultaneously.

As an example, consider spin squeezing by collisional interactions in a two-component Bose Einstein condensate [12, 13]: First, a Bose Einstein condensate is formed in only one of the internal states  $a$ . By a short resonant Raman pulse, all atoms are transferred to an equal superposition of  $a$  and  $b$ . We assume that the interaction strengths between the atoms are not all equal,  $g_{aa} = g_{bb} \neq g_{ab}$ , so that the interaction terms in the fully quantized interaction Hamiltonian

$$\begin{aligned} \hat{H} = & \int d^3r \left\{ \sum_{i=a,b} \left[ \hat{\psi}_i^\dagger(\vec{r}) \hat{h}_i \hat{\psi}_i(\vec{r}) + \frac{g_{ii}}{2} \hat{\psi}_i^\dagger(\vec{r}) \hat{\psi}_i^\dagger(\vec{r}) \hat{\psi}_i(\vec{r}) \hat{\psi}_i(\vec{r}) \right] \right. \\ & \left. + g_{ab} \hat{\psi}_a^\dagger(\vec{r}) \hat{\psi}_b^\dagger(\vec{r}) \hat{\psi}_b(\vec{r}) \hat{\psi}_a(\vec{r}) \right\} \end{aligned} \quad (8)$$

cause a Kerr-like phase evolution of amplitudes on states with different numbers of atoms in the two internal states. This results in squeezing of an appropriate collective spin variable as pointed out by Sørensen et al. [12].

As in our earlier work on this problem [13] we use the *positive P* method [21, 22, 23] to describe the squeezing process. This means that averages of all normal ordered operator products can be calculated as ensemble averages by the substitution of atomic field operators  $\hat{\psi} \equiv (\hat{\psi}_a, \hat{\psi}_a^\dagger, \hat{\psi}_b, \hat{\psi}_b^\dagger)$  by pairs of two-component “wave-functions”,  $\psi = (\psi_{a1}, \psi_{a2}, \psi_{b1}, \psi_{b2})$ . The dynamics of the

wave functions is given by four coupled and noisy “Gross-Pitaevski” equations, see details in Ref. [13]. Starting from a coherent initial state of all atoms in an equal superposition of internal states  $a$  and  $b$ , we numerically simulate solutions of these equations to obtain an ensemble of  $\psi$ ’s describing exactly (up to sampling errors) the quantum correlations of the system.

Once a sizable spin squeezing is obtained we want to transfer this special quantum state to a light pulse, and to be able to use the results of the above analysis we first bring the internal state of the atoms close to the  $a$  state, i.e., with a new resonant Raman pulse we rotate the collective spin close to the north pole of the Bloch sphere. This rotation is applied to the individual sets of “wavefunction” realisations of the simulation ( $i = 1, 2$ ):

$$\begin{pmatrix} \psi_{ai} \\ \psi_{bi} \end{pmatrix} \rightarrow \begin{pmatrix} \psi'_{ai} \\ \psi'_{bi} \end{pmatrix} = \begin{pmatrix} \cos \frac{\theta}{2} & \sin \frac{\theta}{2} \\ -\sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \psi_{ai} \\ \psi_{bi} \end{pmatrix} \quad (9)$$

with  $\theta \cong \pi/2$ .

The atom field interaction in the coupled equations (1) and (2) leads to a natural positive P representation of the field  $\hat{E}_q(z, t)$ : Replace in Eq. (5)  $\hat{E}_q(z, t)$  by a c-number field  $E_{q1}(z, t)$  and  $\psi_b^\dagger(z', 0)\psi_a(z', 0)$  by  $\psi_{b2}(z', 0)\psi_{a1}(z', 0)$ , and make a similar replacement in the hermitian conjugate equation where  $\psi_a^\dagger(z', 0)\psi_b(z', 0)$  is replaced by  $\psi_{a2}(z', 0)\psi_{b1}(z', 0)$  to yield the c-number field  $E_{q2}(z, t)$ . In both cases, the incident vacuum field can be represented by a zero, since the positive P representation yields normally ordered expectation values as simple products. From our original ensemble of quadruples of wavefunctions  $\psi$  describing the atoms just before we turn on  $E_s$  we obtain in this way an ensemble of pairs  $(E_{q1}(z, \tau), E_{q2}(z, \tau))$  describing the generated light field  $\hat{E}_q(z, \tau)$ . With this ensemble any normally ordered field expectation value can be found.

As an example we have investigated 2000 atoms of mass  $m$  and with 1D interaction strengths  $(g_{aa}, g_{ab}, g_{bb} = (1.0, 0.5, 1.0) \times 5 \times 10^{-3} \hbar \Omega l_0$  where  $\Omega$  is the frequency of the harmonic trap and  $l_0 = \sqrt{\hbar/m\Omega}$  is the associated characteristic length. The atoms are spin squeezed by collisional interactions for a time  $t = 3.0\Omega^{-1}$ . They are subsequently driven towards the state  $a$ , and hereafter they are illuminated with the  $E_s$  light which builds up a maximum strength of  $E_{max} = 10^2 \sqrt{2\pi\hbar\omega/l_0}$  in a time of roughly  $t_{rise} = 100l_0/c$ . The matter-light coupling is chosen to be  $\kappa_1 = 10^{-3}c/2\pi\hbar\omega$ .

As we know the atoms to be spin squeezed we also know one mode of the  $\hat{E}_q$  field which will be squeezed: The one corresponding to the simple uniform integral of  $\int \psi_b^\dagger(z', 0)\psi_a(z', 0)dz'$  as described in the discussion after Eq. (6). Even with the simplifications leading to Eqs. (3) and (4) the atomic state could in principle radiate into many other modes and do so with varying quantum statistics. To check whether such other modes are present in this case we calculate the first order correlation function of the field  $\langle \hat{E}_q^\dagger(L, \tau') \hat{E}_q(L, \tau) \rangle =$

$\overline{E_{q2}(L, \tau_2)E_{q1}(L, \tau_1)}$  ( $\overline{\dots}$  indicates averaging over the positive P ensemble). When it is diagonalized we find that almost all population is in fact in the expected mode  $\mathcal{E}(\tau)$  which is plotted in Fig. 1. In other scenarios it might be difficult to calculate beforehand exactly which collective atomic operator is squeezed and then an analysis like this would be necessary in order to pick the local oscillator field for the homodyne detection.

Having confirmed that only one mode is populated we now turn to the quantum character of the field. It depends on how we choose  $\theta$  in Eq. (9): For  $\theta = \pi/2$  it will approximate squeezed vacuum, for  $\theta$  slightly different from  $\pi/2$  it will approximate a squeezed coherent state. As described in [13] the spin squeezing ellipse is at an angle to the coordinate axes. This carries over to the light-field which is squeezed in an appropriate quadrature component  $X_\phi = (\hat{a}e^{i\phi} + \hat{a}^\dagger e^{-i\phi})/\sqrt{2}$ . With the above parameters, we find from the positive P simulations that the minimum variance is  $0.04 \pm 0.01$  corresponding to a reduction by a factor of more than 12 from the standard quantum limit. To illustrate the positive P results in Fig. 2 we show both a histogram and a scatter plot obtained from the positive P ensemble of pairs  $(a_1, a_2)$  representing  $(\hat{a}, \hat{a}^\dagger)$  for the mode of the field.  $a_1$  and  $a_2$ , on average, are complex conjugate quantities, so that the expectation values of hermitian field operators  $\langle X_\phi \rangle = \overline{a_1 e^{i\phi} + a_2 e^{-i\phi}}$  is real for all  $\phi$ . To represent our simulated results we have made a histogram for the values of  $x = X_0$  and  $p = X_{\pi/2}$ , indicated by the *real part* of  $(a_1 + a_2)/\sqrt{2}$  and  $(ia_1 - ia_2)/\sqrt{2}$ . The histogram in Fig. 2 has been obtained from  $10^5$  independent realizations of the noisy Gross-Pitaevskii equation for the atomic spin squeezing, while the scatter plot contains only 6800 points each representing a single realization.

The perfect output coupling of atomic correlations is the main result of the paper, and we imagine that many other atomic quantum states may be taken as starting point for non-classical light generation, e.g., EPR-correlated separated atomic ensembles, and that the mechanism may be used also for reliable interspecies teleportation. A squeezed light pulse may also be applied at the “dark” input port of a beam splitter, to cause the splitting of a strong pulse (with the appropriate mode function  $\mathcal{E}(t)$ ), into two twin pulses with perfectly matched photon statistics. It was recently demonstrated that a classical light pulse can be brought to a complete halt in an atomic sample and that it can be subsequently released [24]. Our study suggests that a quantum light pulse can be similarly stored and retrieved, and that one may perhaps introduce a known or unknown signal field in a condensed sample, manipulate its quantum state by collisional interactions and release it with the original mode function preserved. We emphasize that the spin squeezing mechanism is not crucial for the squeezed light output, and for example the QND-atomic detection and spin squeezing scheme of Kuzmich et al [11] may be

an interesting possibility. Some schemes for atomic spin squeezing may even be compatible with a transmission or production of light with a group velocity so low, that it may constitute a cw source of squeezed light.

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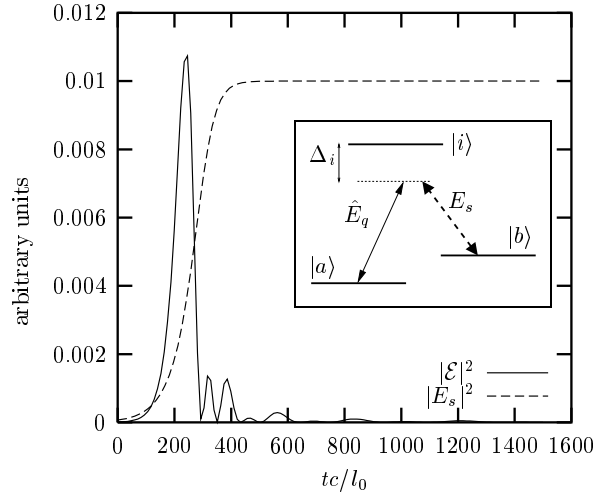


FIG. 1: The shape of the  $E_s$  pulse and the mode function  $\mathcal{E}$  of the emitted  $\hat{E}_q$  pulse. The mode function is found by diagonalizing the first-order correlation function of the field, but coincides with the signal expected when the single atom operator  $\hat{Q}$  is uniform over the sample. The insert shows the atomic level scheme of the proposal.

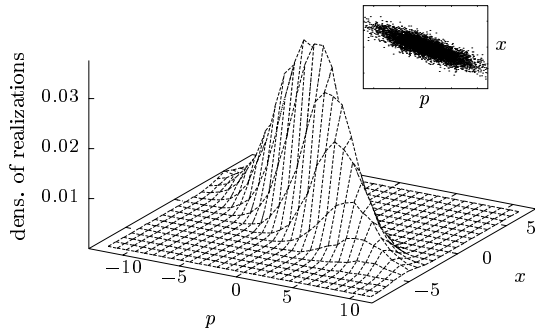


FIG. 2: Histogram illustrating the prediction for the squeezing of light. The quadrature components of the output field are represented by pairs of numbers  $\text{Re}(a_1 + a_2)/\sqrt{2}$ ,  $\text{Re}(ia_1 - ia_2)/\sqrt{2}$ , the distribution of which forms a squeezed ellipsoid shape in phase space. The actual amount of squeezing cannot be directly determined from this plot; it requires a computation of mean values and variances, making use of the fact that the distribution of the complex numbers  $a_1$  and  $a_2$  represent mean values of normally ordered field operators.  $10^5$  realizations contributed to the histogram and in the insert is for illustration shown a scatter plot of 6800 of the representative points.